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# Methodology of Accounting for the Local Surface Heat Exchanges for Investigation of Non-stationary Thermomechanical Processes in the Structure Elements of the Construction

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***Abstract*— The aim of the work was to develop a methodology for taking into account the presence of local surface heat exchanges in rods of finite length, often taking place in studies of the non-stationary phenomenon of thermal conductivity. The proposed methodology was oriented to the subsequent creation of a computational algorithm and its implementation on a personal computer using the universal DELPHI programming tool. This allowed the authors to complete the initial stage of the research works, which will subsequently take into account the internal heat sources in the rods of finite length and constant cross-section in the study of non-stationary thermal conductivity, as well as to develop new methods, approaches and models associated with them.**

***Index Terms*— finite length rod, internal heat sources, local surface heat transfer, non-stationary thermal conductivity, programming tools, and thermomechanical processes.**

## I. Introduction

In mechanical engineering, in particular, in instrument engineering, plastic engineering, and also in other areas, physicotechnical processes often arise, where, any structural element in the form of a rod of finite length is suddenly exposed to thermomechanical loading [1]. Thermomechanical load can be in the form of local temperatures, heat fluxes or heat exchanges, where, in the case of their action on rods of finite length and constant cross-section, non-stationary thermal conductivity can occur in the system, as well as other non-stationary thermoelastic processes [2].

In practice, such phenomena are often encountered at the start of gas-generating, nuclear, hydrogen power plants, rocket and hydrogen engines, as well as internal combustion engines. In transient, unsteady heat conduction processes in the load-bearing elements of these power plants or engines, a complex non-stationary thermally stressed deformed state arises. Herewith, in order to study these processes, many load-bearing elements of the above-mentioned structures can be taken as rods of finite length with a constant cross-section along the entire length. For an adequate description of a non-stationary heat conduction process arising in the rod under the action of dissimilar types of heat sources, taking into account the presence of local thermal insulation, it is necessary to use the classical laws of energy conservation [3], because, the application of energy conservation laws describing such complex non-stationary processes of thermal conductivity in rods of finite length that are exposed to dissimilar kinds of heat sources allows us to take into account natural boundary conditions. As a result, the results obtained will have high accuracy [4]. This, in turn, will contribute to a correct evaluation of the thermally steady-state behavior of the bearing elements [5]. In connection with this, therefore, the development of special effective methods for investigating the non-stationary processes of thermal conductivity of load-bearing elements in the form of rods of finite length and constant cross-section is an actual problem of the applied non-stationary theory of thermoelasticity. At the same time, the development of a complex of applied DELPHI programs that allow studying the classes of non-stationary heat conduction processes for the above systems under the influence of dissimilar types of heat sources, taking into account the presence of local thermal insulation, is of independent scientific interest.

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## II. THE DEVELOPMENT OF THE METHODOLOGY RESULTS AND DISCUSSION

### *A. Algorithm for the construction of local approximating splines of a function*

Based on the known technique [6] consider a horizontal discrete element of a rod of length $l$ [cm] and a constant cross-sectional area $F$ [cm$^2$]. Coefficient of thermal conductivity of the rod material is $k_x$ [W/cm$^o$C]. We direct the horizontal axis $X$, which coincides with the axis of the discrete element of the rod. The calculated scheme of the discrete element of the rod under consideration is shown in Fig. 1.

Suppose that the law of temperature distribution along the length of the discrete element is $T=T(x)$, where $x$ is unknown. In the local coordinate system $(T,x)$, $0 \le x \le l$ we introduce the following notation $T(x=0)=T_i;\ T\left(x=\frac{l}{2}\right)=T_j;\ T(x=l)=T_k$ . These notations are shown in Fig. 2.

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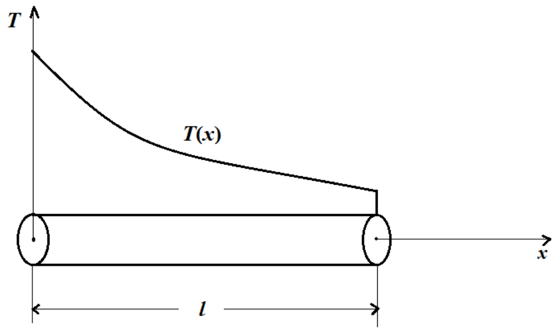


Fig. 1. Calculation scheme of the discrete element of the rod

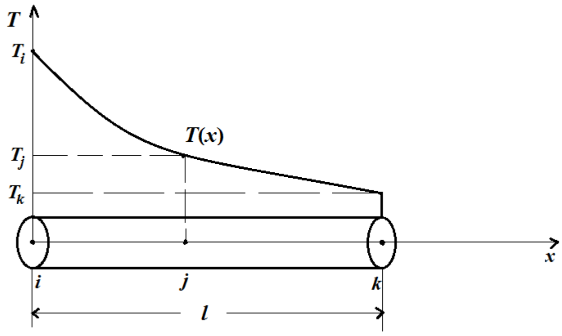


Fig. 2. The design scheme in the notation of the local coordinate system (*T*,*x*)

We denote the node with coordinate $x$=0 by $i$, $x=l/2$ by $j$ and $x=l$ by $k$. n the notation adopted, we can construct the following approximating quadratic function

$$T(x) = ax^2 + bx + c, 0 \le x \le l \qquad (1)$$

*a, b, c* – are some constant values that are not yet known. In addition, also using the adopted notation, we have

$$\left.\begin{aligned} T(x=0) &= a\cdot 0 + b\cdot 0 + c = T_i \\ T\left(x=\tfrac{l}{2}\right) &= a\cdot\left(\tfrac{l}{2}\right)^2 + b\cdot\tfrac{l}{2} + c = T_j \\ T(x=l) &= a\cdot l^2 + b\cdot l + c = T_k \end{aligned}\right\} \qquad (2)$$

Solving this system, we determine the values of the constants *a*, *b*, *c*.

$$c = T_i, \qquad (3)$$

$$\left.\begin{aligned} \frac{al^2}{4} + \frac{bl}{2} &= T_j - T_i \\ al^2 + bl &= T_k - T_i \end{aligned}\right\} \qquad (4)$$

From the last two equations we have $al^2 = T_k - T_i - bl$; $al^2 + 2bl = 4T_j - 4T_i$; or $T_k - T_i - bl + 2bl = 4T_j - 4T_i$; or $bl = 4T_j - 4T_i - T_k + T_i$. Hence we define the value of $b$

$$b = \frac{4T_j - 3T_i - T_k}{l} \qquad (5)$$

Substituting (5) into the second equation of system (4), we obtain

$$al^2 = T_k - T_i + 3T_i + T_k - 4T_j = 2T_k - 4T_j + 2T_i;$$

From this we find the value $a$

$$a = \frac{2T_k - 4T_j + 2T_i}{l^2} \qquad (6)$$

Further, substituting the found values of the constant parameters *a*, *b* and *c* in (1) we obtain

$$T(x) = \frac{2T_k - 4T_j + 2T_i}{l^2}x^2 + \frac{4T_j - 3T_i - T_k}{l}x + T_i, \quad 0 \le x \le l \qquad (7)$$

We simplify equation (7) according to the scheme $(\ldots)T_i + (\ldots)T_j + (\ldots)T_k$

$$T(x) = \left(\frac{2x^2}{l^2} - \frac{3x}{l} + 1\right)T_i + \left(\frac{4x}{l} - \frac{4x^2}{l^2}\right)T_j + \left(\frac{2x^2}{l^2} - \frac{x}{l}\right)T_k = \left(\frac{2x^2 - 3lx + l^2}{l^2}\right)T_i + \left(\frac{4lx - 4x^2}{l^2}\right)T_j + \left(\frac{2x^2 - lx}{l^2}\right)T_k, \quad 0 \le x \le l \qquad (8)$$

Here we introduce the following notation

$$N_i(x) = \frac{2x^2 - 3lx + l^2}{l^2}; \quad N_j(x) = \frac{4lx - 4x^2}{l^2}; \quad N_k(x) = \frac{2x^2 - lx}{l^2}, \qquad (9)$$

Then (8) can be rewritten in the following form

$$T(x) = N_i(x)\cdot T_i + N_j(x)\cdot T_j + N_k(x)\cdot T_k, \quad 0 \le x \le l \qquad (10)$$

It should be noted that the values of the node temperatures $T_i, T_j, T_k$ are still unknown; the functions $N_i(x), N_j(x), N_k(x)$ are approximate quadratic spline functions. Now we study their properties

$$\left.\begin{aligned} &N_i(x=0) = 1; \; N_j(x=0) = 0; \; N_k(x=0) = 0 \\ &N_i\left(x=\tfrac{l}{2}\right) = 0; \; N_j\left(x=\tfrac{l}{2}\right) = 1; \; N_k\left(x=\tfrac{l}{2}\right) = 0 \\ &N_i(x=l) = 0; \; N_j(x=l) = 0; \; N_k(x=l) = 1 \end{aligned}\right\} \qquad (11)$$

From (10) it is also possible to determine the expression for the temperature gradient within the length of the discrete element under consideration.

$$\frac{\partial T}{\partial x} = \frac{\partial N_i(x)}{\partial x}\cdot T_i + \frac{\partial N_j(x)}{\partial x}\cdot T_j + \frac{\partial N_k(x)}{\partial x}\cdot T_k = \frac{4x - 3l}{l^2}\cdot T_i + \frac{4l - 8x}{l^2}\cdot T_j + \frac{4x - l}{l^2}\cdot T_k, \quad 0 \le x \le l \qquad (12)$$

In addition, from (9) it can also be determined that

$$N_i(x) + N_j(x) + N_k(x) = \frac{2x^2 - 3lx + l^2 + 4lx - 4x^2 + 2x^2 - lx}{l^2} = 1 \qquad (13)$$

*B. Development of methods for accounting the presence of local surface heat transfer*

Now consider one discrete element, through the lateral surface of which there is a heat exchange with its surrounding medium. In addition, through the cross-sectional area of the ends of the discrete element of the rod, heat exchange with the surrounding medium also takes place. To begin with, consider the case where the heat transfer coefficient is the same everywhere $h\left[\frac{W}{cm^2\,{}^\circ C}\right]$, the ambient temperature is $T_{amb}[{}^\circ C]$. It is required to construct a resolving system of ordinary differential equations, taking into account the simultaneous presence of local surface heat exchanges. The calculation scheme for the problem under consideration is shown in Fig. 3.

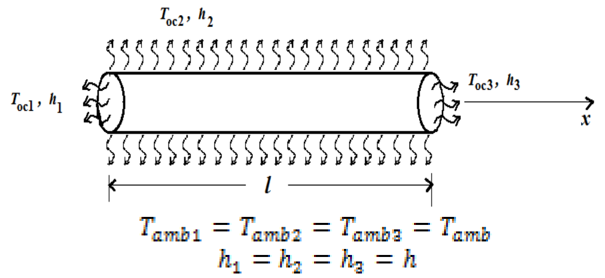


Fig. 3. The design scheme of the problem with surface heat transfer

For the discrete element of the rod under consideration, taking into account the presence of local heat exchanges, we write the expression for the functional of the total thermal energy

$$\Pi = \int_V \frac{k_x}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV + \int_V \lambda \frac{\partial T}{\partial t} T dV + \int_{S(x=0)} \frac{h}{2}(T - T_{oc})^2 dS + \int_{S_{ls}} \frac{h}{2}(T - T_{oc})^2 dS + \int_{S(x=l)} \frac{h}{2}(T - T_{oc})^2 dS \quad (14)$$

Where $V$ is the volume of the discrete element of the rod; $S(x=0)$ is the cross-sectional area of the left end of the discrete element; $S_{ls}$ is the area of the lateral surface of the discrete element of the rod; $S(x=0)$ is the cross-sectional area of the right end of the discrete element; $k_x\left[\frac{W}{°C\cdot cm}\right]$ is the coefficient of thermal conductivity of the material of the rod, and λ is the heat capacity of the material of the rod, whose dimension is $\lambda\left[\frac{W\cdot sec}{cm^3\cdot °C}\right]$.

In equation (14), the first term $\Pi_1 = \int_V \frac{k_x}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV$ – physically means the amount of annulled heat quantity in the volume of the discrete element of the rod; the second term $\Pi_2 = \int_V \lambda \frac{\partial T}{\partial t} T dV$ is the change in the amount of heat per unit time; the other three terms mean the amount of heat that arises from convective heat transfer. Here, it should also be noted that the dimension of Π is $\Pi[W\cdot °C]$ - i.e. the value of the heat work performed. In equation (14), the first term - physically means the amount of annulled heat quantity in the volume of the discrete element of the rod; the second term is the change in the amount of heat per unit time; the other three terms mean the amount of heat that arises from convective heat transfer. Now let`s proceed to integrate all the terms of expression (14). Consider the first term of this functional

$$\Pi_1 = \int_V \frac{k_x}{2}\left(\frac{\partial T}{\partial x}\right)^2 dV = F\int_0^l \frac{k_x}{2}\left(\frac{\partial T}{\partial x}\right)^2 dx,$$

substituting here the expression (12) we have

$$\Pi_1 = \frac{Fk_x}{2}\int_0^l \left[\frac{\partial N_i(x)}{\partial x}T_i + \frac{\partial N_j(x)}{\partial x}T_j + \frac{\partial N_k(x)}{\partial x}T_k\right]^2 dx =$$
$$= \frac{Fk_x}{2l^4}\int_0^l \left[(4x-3l)T_i + (4l-8x)T_j + (4x-l)T_k\right]^2 dx = \quad (15)$$
$$= \frac{Fk_x}{6l}\left(7T_i^2 - 16T_iT_j + 2T_iT_k - 16T_jT_k + 16T_j^2 + 7T_k^2\right)$$

Here the sum of the coefficients will be zero (7-16+2-16+16+7) = 0.

Further we integrate the second integral of the functional (14)

$$\Pi_2 = \int_V \lambda \frac{\partial T}{\partial t} T dV = F\lambda\int_0^l \frac{\partial T}{\partial t}\cdot T dx =$$
$$= F\lambda\int_0^l \left[N_i(x)\frac{\partial T_i}{\partial t} + N_j(x)\frac{\partial T_j}{\partial t} + N_k(x)\frac{\partial T_k}{\partial t}\right]\cdot\left[N_i(x)\cdot T_i + N_j(x)\cdot T_j + N_k(x)\cdot T_k\right]dx =$$
$$= F\lambda\int_0^l \left[N_i^2(x)\cdot T_i\cdot\frac{\partial T_i}{\partial t} + N_i(x)\cdot N_j(x)\cdot T_j\cdot\frac{\partial T_i}{\partial t} + N_i(x)\cdot N_k(x)\cdot T_k\cdot\frac{\partial T_i}{\partial t}\right. \quad (16)$$
$$= N_i(x)\cdot N_j(x)\cdot T_i\cdot\frac{\partial T_j}{\partial t} + N_j^2(x)\cdot T_j\cdot\frac{\partial T_j}{\partial t} + N_j(x)\cdot N_k(x)\cdot T_k\cdot\frac{\partial T_j}{\partial t} +$$
$$\left. + N_i(x)\cdot N_k(x)\cdot T_i\cdot\frac{\partial T_k}{\partial t} + N_j(x)\cdot N_k(x)\cdot T_j\cdot\frac{\partial T_k}{\partial t} + N_k^2(x)\cdot T_k\cdot\frac{\partial T_k}{\partial t}\right]dx$$

In this equation, integrating each integral (of nine) separately and substituting them into (16), we obtain the integrated form of $\Pi_2$;

$$\Pi_2 = F\lambda l\left[\frac{1}{15}T_i\frac{\partial T_i}{\partial t} + \frac{1}{15}T_j\frac{\partial T_i}{\partial t} - \frac{1}{30}T_k\frac{\partial T_i}{\partial t} + \frac{1}{15}T_i\frac{\partial T_j}{\partial t} + \right. \quad (17)$$
$$\left. + \frac{8}{15}T_j\frac{\partial T_j}{\partial t} + \frac{1}{15}T_k\frac{\partial T_j}{\partial t} - \frac{1}{30}T_i\frac{\partial T_k}{\partial t} + \frac{1}{15}T_j\frac{\partial T_k}{\partial t} + \frac{2}{15}T_k\frac{\partial T_k}{\partial t}\right]$$

Now, in the expression (14) we calculate the third integral.

$$\Pi_3 = \int_{S(x=0)} \frac{h}{2}(T - T_{oc})^2 dS = \frac{Fh}{2}(T_i - T_{oc})^2 = \frac{Fh}{2}\left(T_i^2 - 2T_iT_{oc} + T_{oc}^2\right) \quad (18)$$

Similarly, we can calculate the fifth integral in expression (14)

$$\Pi_5 = \int_{S(x=l)} \frac{h}{2}(T - T_{oc})^2 dS = \frac{Fh}{2}(T_k - T_{oc})^2 = \frac{Fh}{2}\left(T_k^2 - 2T_kT_{oc} + T_{oc}^2\right) \quad (19)$$

In the expressions (18)...(19) the sum of the coefficients will be equal to zero (1–2+1) = 0.

The computation of the fourth integral in expression (14) will not be easy

$$\Pi_4 = \int_{S_{пбп}} \frac{h}{2}(T - T_{oc})^2 dS = \frac{Ph}{2}\int_0^l (T - T_{oc})^2 dx = \frac{Ph}{2}\int_0^l \left[(N_i(x)\cdot T_i + N_j(x)\cdot T_j + N_k(x)\cdot T_k) - T_{oc}\right]^2 dx =$$
$$= \frac{Ph}{2}\int_0^l \left[(N_i(x)\cdot T_i + N_j(x)\cdot T_j + N_k(x)\cdot T_k) - T_{oc}\right]\left[(N_i(x)\cdot T_i + N_j(x)\cdot T_j + N_k(x)\cdot T_k) - T_{oc}\right]dx =$$
$$= \frac{Ph}{2}\int_0^l \left[N_i^2(x)\cdot T_i^2 + 2N_i(x)\cdot N_j(x)\cdot T_i\cdot T_j + 2N_i(x)\cdot N_k(x)\cdot T_i\cdot T_k - 2N_i(x)\cdot T_i\cdot T_{oc} + \right. \quad (20)$$
$$\left. + N_j^2(x)\cdot T_j^2 + 2N_j(x)\cdot N_k(x)\cdot T_j\cdot T_k - 2N_j(x)\cdot T_j\cdot T_{oc} + N_k^2(x)\cdot T_k^2 - 2N_k(x)\cdot T_k\cdot T_{oc} + T_{oc}^2\right]dx$$

where P is the perimeter of the cross-section of the discrete element of the rod. Now calculate each integral in expression (20)

$$K_1 = \int_0^l N_i^2(x)\cdot T_i^2 dx = \frac{2l}{15}T_i^2 \quad (21)$$

$$K_2 = \int_0^l 2N_i(x)\cdot N_j(x)\cdot T_i\cdot T_j dx = \frac{2l}{15}T_iT_j \quad (22)$$

$$K_3 = \int_0^l 2N_i(x)\cdot N_k(x)\cdot T_i\cdot T_k dx = -\frac{l}{15}T_iT_k \quad (23)$$

$$K_4=\int_0^l -2N_i(x)\cdot T_i\cdot T_{oc}dx=-\frac{2}{l^2}\int_0^l (2x^2-3lx+l^2)T_iT_{oc}dx=$$
$$=-\frac{2}{l^2}\left(\frac{2x^3}{3}-\frac{3lx^2}{2}+lx\right)\Bigg|_0^l T_iT_{oc}=-\frac{2l^3}{l^2}\left(\frac{4-9+6}{6}\right)T_iT_{oc}=-\frac{l}{3}T_iT_{oc} \quad (24)$$

$$K_5=\int_0^l N_j^2(x)\cdot T_j^2dx=\frac{8l}{15}T_j^2 \quad (25)$$

$$K_6=\int_0^l 2N_j(x)\cdot N_k(x)\cdot T_j\cdot T_kdx=\frac{2l}{15}T_jT_k \quad (26)$$

$$K_7=\int_0^l (-2N_j(x)\cdot T_j\cdot T_{oc})dx=-\frac{2}{l^2}\int_0^l (4lx-4x^2)T_jT_{oc}dx=$$
$$=-\frac{2}{l^2}\left(2lx^2-\frac{4x^3}{3}\right)\Bigg|_0^l T_jT_{oc}=-\frac{2l^3}{l^2}\left(\frac{6-4}{3}\right)T_jT_{oc}=-\frac{4l}{3}T_jT_{oc} \quad (27)$$

$$K_8=\int_0^l N_k^2(x)\cdot T_k^2dx=\frac{1}{l^4}\int_0^l (2x^2-lx)T_k^2dx=\frac{1}{l^4}\int_0^l (4x^4-4lx^3+l^2x^2)T_k^2dx=$$
$$=\frac{1}{l^4}\left(\frac{4x^5}{5}-lx^4+\frac{l^2x^3}{3}\right)\Bigg|_0^l T_k^2=\frac{l}{15}(12-15+5)T_k^2=\frac{2l}{15}T_k^2 \quad (28)$$

$$K_9=\int_0^l (-2N_k(x)\cdot T_k\cdot T_{oc})dx=-\frac{2}{l^2}\int_0^l (2x^2-lx)T_kT_{oc}dx=$$
$$=-\frac{2}{l^2}\left(\frac{2x^3}{3}-\frac{lx^2}{2}\right)\Bigg|_0^l T_kT_{oc}=-\frac{2l^3}{l^2}\left(\frac{4-3}{6}\right)T_kT_{oc}=-\frac{l}{3}T_kT_{oc} \quad (29)$$

$$K_{10}=\int_0^l T_{oc}^2dx=T_{oc}^2\cdot x\big|_0^l=lT_{oc}^2 \quad (30)$$

Now substituting (21)...(30) into (20) we obtain the integrated form $\Pi_4$.

$$\Pi_4=\frac{Phl}{2}\left[\frac{2}{15}T_i^2+\frac{2}{15}T_iT_j-\frac{1}{15}T_iT_k-\frac{1}{3}T_iT_{oc}+\frac{8}{15}T_j^2\right.$$
$$\left.+\frac{2}{15}T_jT_k-\frac{4}{3}T_jT_{oc}+\frac{2}{15}T_k^2-\frac{1}{3}T_kT_{oc}+T_{oc}^2\right] \quad (31)$$

It should be noted here that the sum of the coefficients before the temperatures will be zero

$$\frac{2}{15}+\frac{2}{15}-\frac{1}{15}-\frac{1}{3}+\frac{8}{15}+\frac{2}{15}-\frac{4}{3}+\frac{2}{15}-\frac{1}{3}+1=0$$

Substituting (15), (17) ... (19) and (31) into (14), we find the integrated form of the total thermal energy functional:

$$\Pi=\frac{Fk_x}{6l}\left(7T_i^2-16T_iT_j+2T_iT_k-16T_jT_k+16T_j^2+7T_k^2\right)+$$
$$+F\lambda l\left[\frac{1}{15}T_i\frac{\partial T_i}{\partial t}+\frac{1}{15}T_j\frac{\partial T_i}{\partial t}-\frac{1}{30}T_k\frac{\partial T_i}{\partial t}+\frac{1}{15}T_i\frac{\partial T_j}{\partial t}+\right.$$
$$\left.+\frac{8}{15}T_j\frac{\partial T_j}{\partial t}+\frac{1}{15}T_k\frac{\partial T_j}{\partial t}-\frac{1}{30}T_i\frac{\partial T_k}{\partial t}+\frac{1}{15}T_j\frac{\partial T_k}{\partial t}+\frac{2}{15}T_k\frac{\partial T_k}{\partial t}\right]+$$
$$+\frac{Fh}{2}\left(T_i^2-2T_iT_{oc}+T_{oc}^2\right)+\frac{Fh}{2}\left(T_k^2-2T_kT_{oc}+T_{oc}^2\right)+$$
$$+\frac{Phl}{2}\left[\frac{2}{15}T_i^2+\frac{2}{15}T_iT_j-\frac{1}{15}T_iT_k-\frac{1}{3}T_iT_{oc}+\frac{8}{15}T_j^2+\frac{2}{15}T_jT_k\right.$$
$$\left.-\frac{4}{3}T_jT_{oc}+\frac{2}{15}T_k^2-\frac{1}{3}T_kT_{oc}+T_{oc}^2\right] \quad (32)$$

Further, minimizing the functional $\Pi$ with respect to the node values $T_i$, $T_j$ *and* $T_k$ we obtain a resolving system of first-order ordinary differential equations with the corresponding initial conditions and taking into account existing local heat exchanges.

$$\frac{\partial\Pi}{\partial T_i}=0;\rightarrow\frac{Fk_x}{6l}\left(14T_i-16T_j+2T_k\right)+F\lambda l\left(\frac{1}{15}\frac{\partial T_i}{\partial t}+\frac{1}{15}\frac{\partial T_j}{\partial t}-\frac{1}{30}\frac{\partial T_k}{\partial t}\right)+\frac{Fh}{2}\left(2T_i-2T_{oc}\right)+\frac{Phl}{2}\left(\frac{4}{15}T_i+\frac{2}{15}T_j-\frac{1}{15}T_k-\frac{1}{3}T_{oc}\right)=0;$$

$$\frac{\partial\Pi}{\partial T_j}=0;\rightarrow\frac{Fk_x}{6l}\left(-16T_i+32T_j-16T_k\right)+F\lambda l\left(\frac{1}{15}\frac{\partial T_i}{\partial t}+\frac{8}{15}\frac{\partial T_j}{\partial t}+\frac{1}{15}\frac{\partial T_k}{\partial t}\right)+\frac{Phl}{2}\left(\frac{2}{15}T_i+\frac{16}{15}T_j+\frac{2}{15}T_k-\frac{4}{3}T_{oc}\right)=0;$$

$$\frac{\partial\Pi}{\partial T_k}=0;\rightarrow\frac{Fk_x}{6l}\left(2T_i-16T_j+14T_k\right)+F\lambda l\left(-\frac{1}{30}\frac{\partial T_i}{\partial t}+\frac{1}{15}\frac{\partial T_j}{\partial t}+\frac{2}{15}\frac{\partial T_k}{\partial t}\right)+\frac{Fh}{2}\left(2T_k-2T_{oc}\right)+\frac{Phl}{2}\left(-\frac{1}{15}T_i+\frac{2}{15}T_j+\frac{4}{15}T_k-\frac{1}{3}T_{oc}\right)=0.$$

Or, after simplification from the last system, we get:

$$\begin{cases}\dfrac{\partial T_i}{\partial t}+\dfrac{\partial T_j}{\partial t}-\dfrac{1}{2}\dfrac{\partial T_k}{\partial t}+\left(\dfrac{35k_x}{\lambda l^2}+\dfrac{15h}{\lambda l}+\dfrac{2Ph}{F\lambda}\right)T_i \\ \quad+\left(\dfrac{Ph}{F\lambda}-\dfrac{40k_x}{\lambda l^2}\right)T_j+\left(\dfrac{5k_x}{\lambda l^2}+\dfrac{Ph}{2F\lambda}\right)T_k \\ \quad=\dfrac{15hT_{oc}}{\lambda l}+\dfrac{5PhT_{oc}}{2F\lambda}\dfrac{\partial T_i}{\partial t}+8\dfrac{\partial T_j}{\partial t}+\dfrac{\partial T_k}{\partial t} \\ \quad+\left(\dfrac{Ph}{F\lambda}-\dfrac{40k_x}{\lambda l^2}\right)T_i+\left(\dfrac{80k_x}{\lambda l^2}+\dfrac{8Ph}{F\lambda}\right)T_j+ \\ \quad\left(\dfrac{Ph}{F\lambda}-\dfrac{80k_x}{\lambda l^2}\right)T_k=\dfrac{10PhT_{oc}}{F\lambda}-\dfrac{1}{2}\dfrac{\partial T_i}{\partial t}+ \\ \quad+\dfrac{\partial T_j}{\partial t}+2\dfrac{\partial T_k}{\partial t}+\left(\dfrac{5k_x}{\lambda l^2}-\dfrac{Ph}{2F\lambda}\right)T_i \\ +\left(\dfrac{Ph}{F\lambda}-\dfrac{40k_x}{\lambda l^2}\right)T_j+\left(\dfrac{35k_x}{\lambda l^2}+\dfrac{15h}{\lambda l}+\dfrac{2Ph}{F\lambda}\right)T_k=\dfrac{5PhT_{oc}}{F\lambda}\end{cases} \quad (33)$$

Setting the initial conditions

$$T_i\big|_{t=0}=a_1;\ T_j\big|_{t=0}=a_2;\ T_k\big|_{t=0}=a_3$$

one can find a solution of the system (33).

The obtained results are oriented for the computational algorithm created by the authors, which was realized in a personal computer in the form of Delphi programs [7]. With

the help of the Delphi-7 programming developed on the object-oriented programming language, we solved the non-stationary heat conduction problem for a rod of finite length under the influence of local heat transfer along the lateral surface [8].

## III. CONCLUSION

Based on the energy conservation laws, a technique is developed that simultaneously takes into account the presence of local surface heat exchanges for the non-stationary heat conduction problem occurring in a rod of finite length and the constancy of its cross-sectional area. Applying the quadratic approximation function of the form from the energy conservation law by the method of minimizing it through the node temperature values, we obtain solving systems of first-order linear ordinary equations that take into account the natural boundary conditions. Due to this, the results obtained have a high degree of accuracy. The developed methodology and the corresponding computational algorithm allowed to realize calculations in a personal computer on the object-oriented Delphi-7 programming language, and thus to solve the non-stationary heat conduction problem for a rod of finite length under the influence of local heat exchange along the lateral surface.

## REFERENCES

[1] Maugin, O.A. “The Thermomechanics of Nonlinear Irreversible Behaviors: An Introduction”, Singapore: World Scientific, 1999, 375 p.

[2] Pantusoa, D., Klaus-Juergen Bathe, Bouzinov, P.A. “A finite element procedure for the analysis of thermo-mechanical solids in contact”, Computers and Structures 75, 2000, Vol. No. 2, pp. 551-573.

[3] Belytschko T, Guo, У., Liu, W., Xiao S., “A unified stability analysis of meshless particle methods”, International Journal for Numerical Methods in Engineering, 2000, No. 48 (9), pp. 1359-1400.

[4] Deang, J., Du., Q., and Gunzburger, M., “Modeling and computation of random thermal f1uctuations and material defects in the Ginzburg-Landau model for superconductivity”, Journal of Computational Physics, 2002, Vol. 181, 2001, pp. 45-67.

[5] Gurtin, M.E., “An Introduction to Continuum Mechanics”, New York: Academic Press, 1981, 830 p.

[6] Biswajit, B.A., “Material Point Method Formulation for Plasticity”, Computational Physics, 2006, Vol. 51, pp. 1-25.

[7] Tashenova, Zh., Uzakkyzy, N., Omarkhanova, D., Nurlybaeva, E., Kudaykulov. A., “Algorithm for calculation of parameters of the bearing elements of oil heating installation”, IMPACT: International Journal of Computational Sciences and Information Technology, 2015, Vol. No. 1, Issue 1, pp. 13-19.

[8] Tashenova, Zh., Nurlybaeva, E., Kudaykulov, A.K., Tashatov, N., “Modeling of Thermo-Stressed State of Heat-Resistant Alloy Rod in the Presence of Temperature Constant Intensity”, Computer Sciences and Application, 2015. Vol. 2, No. 4, pp. 157-160.

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